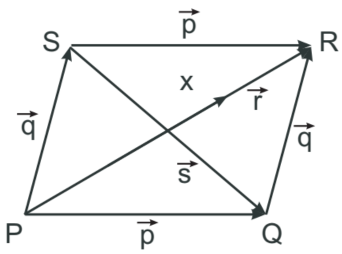
Vectors

Case Study Based Questions

Case Study 1

PQRS is a parallelogram whose adjacent sides are represented by the vectors $\vec{p}$ and $\vec{q}$. Three of its vertices are *P* (4, −2, 1), *Q* (3, −1 , 0) and *S* (1, −1 , −1).



Based on the above information, solve the following questions:

Q 1. The vector $\vec{p} + \vec{q}$ is:

a. $-4\hat{i} + 2\hat{j} - 3\hat{k}$ b. $4\hat{i} - 2\hat{j} - 3\hat{k}$

c. $-4\hat{i} + 2\hat{j} + 3\hat{k}$ d. $-\hat{i} + \hat{j} + \hat{k}$

Q 2. A unit vector along the vector $(\vec{p} + \vec{q})$ is:

a. $\frac{2\hat{i} - \hat{j} + \hat{k}}{\sqrt{6}}$ b. $\frac{-4\hat{i} + 2\hat{j} - 3\hat{k}}{\sqrt{29}}$

c. $\frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$ d. $\frac{\hat{i} - 2\hat{k}}{\sqrt{5}}$

Q 3. The diagonal $\vec{s}$ is:

a. $(2\hat{i} - \hat{j})$ b. $(\hat{j} + 2\hat{k})$ c. $(2\hat{i} + \hat{k})$ d. $(\hat{i} - \hat{k})$

Q 4. Area of *PQRS*, whose adjacent sides are $\vec{p}$ and $\vec{q}$, is:

a. $\sqrt{2}$ sq. units b. $\sqrt{3}$ sq. units

c. $\sqrt{5}$ sq. units d. $\sqrt{6}$ sq. units

Q 5. The value of $\frac{1}{2}|\vec{r} \times \vec{s}|$ is:

a. $\sqrt{6}$ b. $2\sqrt{2}$ c. $\sqrt{3}$ d. $\sqrt{5}$

Solutions

1. Position vector of the points *P*, *Q* and *S* are

$$\overrightarrow{OP} = 4\hat{i} - 2\hat{j} + \hat{k},\ \overrightarrow{OQ} = 3\hat{i} - \hat{j}$$

and $$\overrightarrow{OS} = \hat{i} - \hat{j} - \hat{k}$$

$\therefore$ $$\vec{p} = \overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$= (3\hat{i} - \hat{j}) - (4\hat{i} - 2\hat{j} + \hat{k})$$

$$= -\hat{i} + \hat{j} - \hat{k}$$

and $$\vec{q} = \vec{PS} = \vec{OS} - \vec{OP}$$

$$= (\hat{i} - \hat{j} - \hat{k}) - (4\hat{i} - 2\hat{j} + \hat{k})$$

$$= -3\hat{i} + \hat{j} - 2\hat{k}$$

Now, $$\vec{p} + \vec{q} = (-\hat{i} + \hat{j} - \hat{k}) + (-3\hat{i} + \hat{j} - 2\hat{k})$$

$$= -4\hat{i} + 2\hat{j} - 3\hat{k}$$

So, option (a) is correct.

2. From part (1), $\vec{p} + \vec{q} = -4\hat{i} + 2\hat{j} - 3\hat{k}$

Now, $$|\vec{p} + \vec{q}| = |-4\hat{i} + 2\hat{j} - 3\hat{k}|$$

$$= \sqrt{(-4)^2 + (2)^2 + (-3)^2}$$

$$= \sqrt{16 + 4 + 9} = \sqrt{29}$$

$\therefore$ A unit vector along the vector $(\vec{p} + \vec{q}) = \dfrac{(\vec{p} + \vec{q})}{|\vec{p} + \vec{q}|}$

$$= \frac{-4\hat{i} + 2\hat{j} - 3\hat{k}}{\sqrt{29}}$$

So, option (b) is correct.

3. Diagonal $\vec{s} = \vec{p} - \vec{q} = (-\hat{i} + \hat{j} - \hat{k}) - (-3\hat{i} + \hat{j} - 2\hat{k})$

$$= 2\hat{i} + \hat{k}$$

So, option (c) is correct.

4. Now, $\vec{p} \times \vec{q} = (-\hat{i} + \hat{j} - \hat{k}) \times (-3\hat{i} + \hat{j} - 2\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -1 \\ -3 & 1 & -2 \end{vmatrix}$$

$$= \hat{i}(-2+1) - \hat{j}(2-3) + \hat{k}(-1+3)$$

$$= -\hat{i} + \hat{j} + 2\hat{k}$$

$\therefore$ Area of parallelogram $PQRS = |\vec{p} \times \vec{q}|$

$$= |-\hat{i} + \hat{j} + 2\hat{k}| = \sqrt{(-1)^2 + (1)^2 + (2)^2}$$

$$= \sqrt{1+1+4} = \sqrt{6} \text{ sq. units}$$

So, option (d) is correct.

5. Diagonal $\vec{r} = \vec{p} + \vec{q} = -4\hat{i} + 2\hat{j} - 3\hat{k}$

[from part (1)]

and diagonal $\vec{s} = 2\hat{i} + \hat{k}$ [from part (3)]

Now, $\vec{r} \times \vec{s} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 2 & -3 \\ 2 & 0 & 1 \end{vmatrix}$

$$= \hat{i}(2-0) - \hat{j}(-4+6) + \hat{k}(0-4)$$

$$= 2\hat{i} - 2\hat{j} - 4\hat{k}$$

and $|\vec{r} \times \vec{s}| = |2\hat{i} - 2\hat{j} - 4\hat{k}|$

$$= \sqrt{(2)^2 + (-2)^2 + (-4)^2}$$

$$= \sqrt{4+4+16}$$

$$= \sqrt{24} = 2\sqrt{6}$$

$\therefore$ $\frac{1}{2}|\vec{r} \times \vec{s}| = \sqrt{6}$

So, option (a) is correct.

Case Study 2

Students of Class-XII appearing for a class test of Mathematics. The questions of test paper is based on vector algebra. All students were asked to attempt the following questions:

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors.

Based on the above information, solve the following questions:

Q 1. The position vector of the point which divides the join of points with position vectors $\vec{a}+2\vec{b}$ and $\vec{a}-2\vec{b}$ in the ratio 2 : 3 is:

a. $\frac{5\vec{a}+2\vec{b}}{5}$

b. $\frac{2\vec{a}+5\vec{b}}{5}$

c. $\frac{2\vec{a}+3\vec{b}}{5}$

d. $\frac{3\vec{a}+2\vec{b}}{5}$

Q 2. The projection of vector $\vec{a}=\hat{i}-3\hat{j}+2\hat{k}$ along $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$ is:

a. $\frac{2}{3}$

b. $\frac{1}{\sqrt{14}}$

c. $\sqrt{7}$

d. $\frac{1}{\sqrt{7}}$

Q 3. The vector in the direction of the vector $3\,i+4\,k$ that has magnitude 25, is:

a. $\frac{(3\hat{i}+4\hat{k})}{5}$

b. $(3\hat{i}+4\hat{k})$

c. $5(3\hat{i}+4\hat{k})$

d. $\frac{3\hat{i}+4\hat{k}}{25}$

Q 4. The value of λ such that the vectors $\vec{a}=\hat{i}-2\hat{j}+\lambda\hat{k}$ and $\vec{b}=3\hat{i}+\hat{j}-\hat{k}$ are orthogonal, is:

a. 4

b. 3

c. 2

d. 1

Q 5. The vectors from origin to the points *A* and *B* are $\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$ respectively, then the area of $\triangle OAB$ is:

a. 340 sq. units

b. $\sqrt{255}$ sq. units

c. $\sqrt{229}$ sq. units

d. $\frac{1}{2}\sqrt{229}$ sq. units

Solutions

1. Position vector of the required point

$$= \frac{3(\vec{a} + 2\vec{b}) + 2(\vec{a} - 2\vec{b})}{2+3}$$

$$= \frac{3\vec{a} + 6\vec{b} + 2\vec{a} - 4\vec{b}}{5} = \frac{5\vec{a} + 2\vec{b}}{5}$$

So, option (a) is correct.

2. Projection of a vector $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$$= \frac{(\hat{i} - 3\hat{j} + 2\hat{k}) \cdot (\hat{i} + 2\hat{j} + 3\hat{k})}{|\hat{i} + 2\hat{j} + 3\hat{k}|}$$

$$= \frac{(1)(1) + (-3)(2) + (2)(3)}{\sqrt{(1)^2 + (2)^2 + (3)^2}} = \frac{1 - 6 + 6}{\sqrt{1 + 4 + 9}} = \frac{1}{\sqrt{14}}$$

So, option (b) is correct.

3. Let $\vec{a} = 3\hat{i} + 4\hat{k} = 3\hat{i} + 0\hat{j} + 4\hat{k}$

and $|\vec{a}| = \sqrt{(3)^2 + (0)^2 + (4)^2}$

$$= \sqrt{9 + 0 + 16} = \sqrt{25} = 5$$

$\therefore$ The vector in the direction of $\vec{a}$ that has magnitude 25

$$= 25 \times \frac{\vec{a}}{|\vec{a}|} = 25 \times \frac{(3\hat{i} + 4\hat{k})}{5} = 5(3\hat{i} + 4\hat{k})$$

So, option (c) is correct.

4. Given, $\vec{a} = \hat{i} - 2\hat{j} + \lambda\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$.

Since, $\vec{a}$ and $\vec{b}$ are orthogonal.

$$\therefore \quad \vec{a}\cdot\vec{b} = 0$$

$$\Rightarrow \quad (\hat{i} - 2\hat{j} + \lambda\hat{k})\cdot(3\hat{i} + \hat{j} - \hat{k}) = 0$$

$$\Rightarrow \quad (1)(3) + (-2)(1) + (\lambda)(-1) = 0$$

$$\Rightarrow \quad 3 - 2 - \lambda = 0 \Rightarrow \lambda = 1$$

So, option (d) is correct.

5. Given, $\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$

$$\text{Now,} \quad \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \hat{i}(-3-6) - \hat{j}(2-4) + \hat{k}(6+6)$$

$$= -9\hat{i} + 2\hat{j} + 12\hat{k}$$

$$\text{and} \quad |\vec{a}\times\vec{b}| = \sqrt{(-9)^2 + (2)^2 + (12)^2}$$

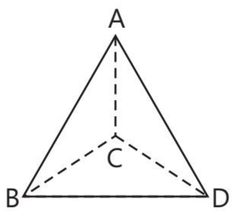
$$= \sqrt{81 + 4 + 144} = \sqrt{229}$$

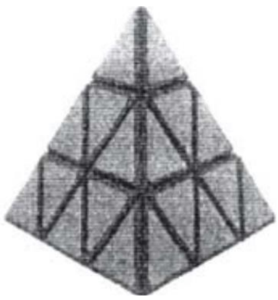
$$\therefore \text{ Area of } \Delta OAB = \frac{1}{2}|\vec{a}\times\vec{b}| = \frac{1}{2}\sqrt{229}$$

So, option (d) is correct.

Case Study 3

A building is to be constructed in the form of a triangular pyramid ABCD as shown in the figure.





Let its angular points be A (0, 1, 2), B (3, 0, 1), C (4, 3, 6) and D (2, 3, 2) and G be the point of intersection of the medians of ABCD.

Based on the above information, solve the following questions:

Q 1. The coordinates of point G are:

a. (2, 3, 3) b. (3, 3, 2) c. (3, 2, 3) d. (0, 2, 3)

Q 2. The length of vector $\overrightarrow{AG}$ is:

a. $\sqrt{17}$ units b. $\sqrt{11}$ units c. $\sqrt{13}$ units d. $\sqrt{19}$ units

Q 3. Area of ΔABC (in sq. units) is:

a. $\sqrt{10}$ b. $2\sqrt{10}$ c. $3\sqrt{10}$ d. $5\sqrt{10}$

Q 4. The sum of lengths of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is:

a. 5 units b. 9.32 units c. 10 units d. 11 units

Q 5. The length of the perpendicular from the vertex D on the opposite face is:

a. $\frac{6}{\sqrt{10}}$ units b. $\frac{2}{\sqrt{10}}$ units

c. $\frac{3}{\sqrt{10}}$ units d. $8\sqrt{10}$ units

Solutions

1.

Clearly, G is the centroid of ΔBCD, therefore coordinate of G are

$$\left(\frac{3+4+2}{3}, \frac{0+3+3}{3}, \frac{1+6+2}{3}\right) = (3, 2, 3)$$

So, option (c) is correct.

2. Since, $A \equiv (0, 1, 2)$ and $G \equiv (3, 2, 3)$

$$\therefore \quad \overrightarrow{AG} = (3-0)\hat{i} + (2-1)\hat{j} + (3-2)\hat{k}$$

$$= 3\hat{i} + \hat{j} + \hat{k}$$

$$\Rightarrow |\overrightarrow{AG}|^2 = 3^2 + 1^2 + 1^2 = 9 + 1 + 1 = 11$$

$$\Rightarrow |\overrightarrow{AG}| = \sqrt{11} \text{ units}$$

So, option (b) is correct.

3. Clearly, area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

Here, $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3-0 & 0-1 & 1-2 \\ 4-0 & 3-1 & 6-2 \end{vmatrix}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -1 \\ 4 & 2 & 4 \end{vmatrix}$$

$$= \hat{i}(-4+2) - \hat{j}(12+4) + \hat{k}(6+4)$$

$$= -2\hat{i} - 16\hat{j} + 10\hat{k}$$

$$\therefore \quad |\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-2)^2 + (-16)^2 + 10^2}$$

$$= \sqrt{4 + 256 + 100}$$

$$= \sqrt{360} = 6\sqrt{10}$$

Hence, area of $\Delta ABC = \frac{1}{2} \times 6\sqrt{10} = 3\sqrt{10}$ sq. units

So, option (c) is correct.

4. Here, $\overrightarrow{AB} = 3\hat{i} - \hat{j} - \hat{k}$

$$\Rightarrow \quad |\overrightarrow{AB}| = \sqrt{9+1+1} = \sqrt{11}$$

Also, $\overrightarrow{AC} = 4\hat{i} + 2\hat{j} + 4\hat{k}$

$$\Rightarrow \quad |\overrightarrow{AC}| = \sqrt{16+4+16} = \sqrt{36} = 6$$

Now, $|\overrightarrow{AB}| + |\overrightarrow{AC}| = \sqrt{11} + 6 = 3.32 + 6 = 9.32$ units

So, option (b) is correct.

5. The length of the perpendicular from the vertex D on the opposite face

$$= |\text{Projection of } \overrightarrow{AD} \text{ on } \overrightarrow{AB} \times \overrightarrow{AC}|$$

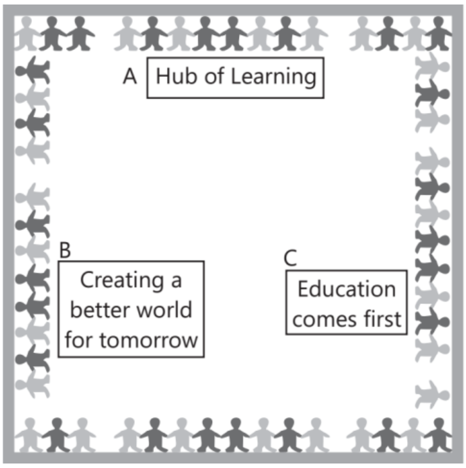
$$= \left| \frac{(2\hat{i} + 2\hat{j}) \cdot (-2\hat{i} - 16\hat{j} + 10\hat{k})}{\sqrt{(-2)^2 + (-16)^2 + 10^2}} \right|$$

$$= \left| \frac{-4-32}{\sqrt{360}} \right| = \frac{36}{6\sqrt{10}} = \frac{6}{\sqrt{10}} \text{ units}$$

So, option (a) is correct.

Case Study 4

Three slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (Hub of Learning), B (Creating a better world for tomorrow) and C (Education comes first). The coordinates of these points are (1, 4, 2), (3, −3, −2) and (-2, 2, 6) respectively.



Based on the given information, solve the following questions:

Q 1. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the position vectors of points *A*, *B* and *C* respectively, then $\vec{a}+\vec{b}+\vec{c}$ is equal to:

a. $2\hat{i}+3\hat{j}+6\hat{k}$ b. $2\hat{i}-3\hat{j}-6\hat{k}$

c. $2\hat{i}+8\hat{j}+3\hat{k}$ d. $2(7\hat{i}+8\hat{j}+3\hat{k})$

Q 2. Which of the following is not true?

a. $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$ b. $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{AC}=\vec{0}$

c. $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{CA}=\vec{0}$ d. $\overrightarrow{AB}-\overrightarrow{CB}+\overrightarrow{CA}=\vec{0}$

Q 3. Area of ΔABC is:

a. 19 sq. units b. $\sqrt{1937}$ sq. units

c. $\frac{1}{2}\sqrt{1937}$ sq. units d. $\sqrt{1837}$ sq. units

Q 4. Suppose, if the given slogans are to be placed on a straight line, then the value of $|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|$ will be equal to:

a. -1 b. -2 c. 2 d. 0

Q 5. If $\vec{a}=2\hat{i}+3\hat{j}+6\hat{k}$, then unit vector in the direction of vector $\vec{a}$ is:

a. $\frac{2}{7}\hat{i}-\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}$ b. $\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{6}{7}\hat{k}$

c. $\frac{3}{7}\hat{i}+\frac{2}{7}\hat{j}+\frac{6}{7}\hat{k}$ d. None of these

Solutions

1. $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 3\hat{j} - 2\hat{k}$

and $\vec{c} = -2\hat{i} + 2\hat{j} + 6\hat{k}$

$\therefore$ $\vec{a} + \vec{b} + \vec{c} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

So, option (a) is correct.

2. Using triangle law of addition in ΔABC, we get

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$, which can be rewritten as

$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$ or $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

So, option (c) is correct.

3. We have, $A(1, 4, 2), B(3, -3, -2)$ and $C(-2, 2, 6)$

Now, $\overrightarrow{AB} = \vec{b} - \vec{a} = 2\hat{i} - 7\hat{j} - 4\hat{k}$

and $\overrightarrow{AC} = \vec{c} - \vec{a} = -3\hat{i} - 2\hat{j} + 4\hat{k}$

$$\therefore \quad \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & -4 \\ -3 & -2 & 4 \end{vmatrix}$$

$$= \hat{i}(-28 - 8) - \hat{j}(8 - 12) + \hat{k}(-4 - 21)$$

$$= -36\hat{i} + 4\hat{j} - 25\hat{k}$$

Now, $|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-36)^2 + 4^2 + (-25)^2}$

$$= \sqrt{1296 + 16 + 625} = \sqrt{1937}$$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{1937}$ sq. units

So, option (c) is correct.

4. If the given points lie on the straight line, then the points will be collinear and so area of $\Delta ABC = 0$.

$\Rightarrow$ $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}| = 0$

[$\because$ If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the three vertices A, B and C of ΔABC, then area of triangle $= \frac{1}{2}[|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|]$]

So, option (d) is correct.

5. Here, $|\vec{a}| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36}$

$= \sqrt{49} = 7$

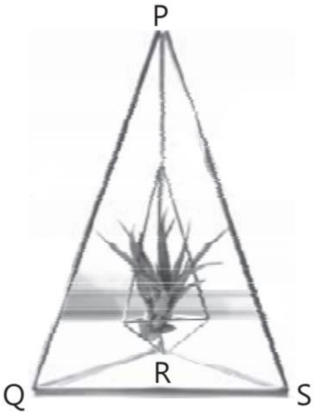
$\therefore$ Unit vector in the direction of vector $\vec{a}$ is

$$\hat{a} = \frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{7} = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$

So, option (b) is correct.

Case Study 5

Rakesh purchased an air plant holder which is in the shape of a tetrahedron. Let P, Q, R and S be the coordinates of the air plant holder where $P \equiv (3, 3, 4)$, $Q \equiv (3, 1, 2)$, $R \equiv (2, 1, 3)$ and $S \equiv (1, 1, 1)$.



Based on the above information, solve the following questions.

Q 1. Find the position vector of $\overrightarrow{PS}$.

Q 2. Find the area of ΔPQR.

Q 3. Find the unit vector along $\overrightarrow{PS}$.

Or

Find the projection of $\overrightarrow{PQ}$ on $\overrightarrow{PR}$.

Solutions

1. Here, $\overrightarrow{OP} = 3\hat{i} + 3\hat{j} + 4\hat{k}$ and $\overrightarrow{OS} = \hat{i} + \hat{j} + \hat{k}$

$\therefore$ Position vector of $\overrightarrow{PS} = \overrightarrow{OS} - \overrightarrow{OP}$

$= (\hat{i} + \hat{j} + \hat{k}) - (3\hat{i} + 3\hat{j} + 4\hat{k}) = -2\hat{i} - 2\hat{j} - 3\hat{k}$

2. Here, $\overrightarrow{OQ} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{OR} = 2\hat{i} + \hat{j} + 3\hat{k}$

Now, position vector of $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= (3\hat{i} + \hat{j} + 2\hat{k}) - (3\hat{i} + 3\hat{j} + 4\hat{k}) = -2\hat{j} - 2\hat{k}$

and position vector of $\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP}$

$$= (2\hat{i} + \hat{j} + 3\hat{k}) - (3\hat{i} + 3\hat{j} + 4\hat{k})$$

$$= -\hat{i} - 2\hat{j} - \hat{k}$$

Now, $\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -2 & -2 \\ -1 & -2 & -1 \end{vmatrix}$

$$= \hat{i}(2-4) - \hat{j}(0-2) + \hat{k}(0-2)$$

$$= -2\hat{i} + 2\hat{j} - 2\hat{k}$$

$\Rightarrow \quad |\overrightarrow{PQ} \times \overrightarrow{PR}| = \sqrt{(-2)^2 + (2)^2 + (-2)^2}$

$$= \sqrt{4+4+4} = 2\sqrt{3}$$

$\therefore$ Area of $\Delta PQR = \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}|$

$$= \frac{1}{2} \times 2\sqrt{3} = \sqrt{3} \text{ sq. units}$$

3. Unit vector along $\overrightarrow{PS} = \frac{\overrightarrow{PS}}{|\overrightarrow{PS}|}$

$$= \frac{-2\hat{i} - 2\hat{j} - 3\hat{k}}{\sqrt{(-2)^2 + (-2)^2 + (-3)^2}}$$

$$= \frac{-2\hat{i} - 2\hat{j} - 3\hat{k}}{\sqrt{4+4+9}} = -\frac{1}{\sqrt{17}}(2\hat{i} + 2\hat{j} + 3\hat{k})$$

Or

Here, $\overrightarrow{PR} = -\hat{i} - 2\hat{j} - \hat{k}$

and $\overrightarrow{PQ} = -2\hat{j} - 2\hat{k}$

$\therefore$ Projection of $\overrightarrow{PQ}$ on $\overrightarrow{PR} = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{|\overrightarrow{PR}|}$

$$= \frac{(-\hat{i} - 2\hat{j} - \hat{k}) \cdot (-2\hat{j} - 2\hat{k})}{|-\hat{i} - 2\hat{j} - \hat{k}|}$$

$$= \frac{(-2)(-2) + (-2)(-1)}{\sqrt{(-1)^2 + (-2)^2 + (-1)^2}}$$

$$= \frac{4+2}{\sqrt{1+4+1}} = \frac{6}{\sqrt{6}} = \sqrt{6}$$

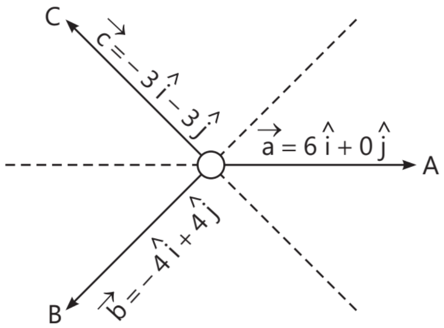
Case Study 6

Teams A, B, C went for playing a tug of war game. Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area.

Team A pulls with force $F_1 = 6\hat{i} + 0\hat{j}$ kN,

Team B pulls with force $F_2 = -4\hat{i} + 4\hat{j}$ kN,

Team C pulls with force $F_3 = -3\hat{i} - 3\hat{j}$ kN



Based on the above information, solve the following questions: (CBSE SQP 2023-24)

Q1. What is the magnitude of the force of Team A?

Q2. Which team will win the game?

Q3. Find the magnitude of the resultant force exerted by the teams.

Or

In what direction is the ring getting pulled?

Solutions

1. The magnitude of the force of team $A = |\vec{F_1}|$

$$= |6\hat{i} + 0\hat{j}| = \sqrt{6^2 + 0} = 6 \text{ kN}$$

2. Since, $|\vec{F_1}| = 6$ kN

Now, $|\vec{F_2}| = |-4\hat{i} + 4\hat{j}| = \sqrt{(-4)^2 + (4)^2}$

$= \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$ kN

and $|\vec{F_3}| = |-3\hat{i} - 3\hat{j}| = \sqrt{(-3)^2 + (-3)^2}$

$= \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$ kN

Since, 6 is larger, so team A wins.

3. The magnitude of the resultant force,

$$|\vec{F}| = |\vec{F_1} + \vec{F_2} + \vec{F_3}|$$

$$= |(6\hat{i} + 0\hat{j}) + (-4\hat{i} + 4\hat{j}) + (-3\hat{i} - 3\hat{j})|$$

$$= |-\hat{i} + \hat{j}| = \sqrt{(-1)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}$$

Or

We have, $\vec{F} = -\hat{i} + \hat{j}$

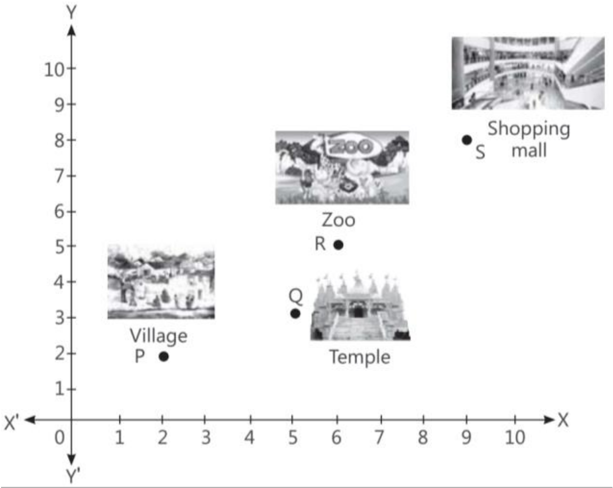
$$\therefore \quad \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{1}{-1}\right)$$

$$= \tan^{-1}(-1) = -\tan^{-1}(1)$$

$$= -\frac{\pi}{4}$$

Case Study 7

Tanya left from her village on weekend. First, she travelled up to temple. After this, she left for the zoo. After this she left for shopping in a mall. The positions of Tanya at different places is given in the following graph.



Based on the above information, solve the following questions:

Q 1. Find the vector $\overrightarrow{QR}$ in terms of $\hat{i}, \hat{j}$.

Q 2. Find the length of vector $\overrightarrow{PS}$.

Q 3. Find the unit vector of $\overrightarrow{PR}$.

Or

Find $\overrightarrow{PR} \times \overrightarrow{QS}$.

Solutions

1. Position vector of $Q = 5\hat{i} + 3\hat{j}$

and position vector of $R = 6\hat{i} + 5\hat{j}$

$\therefore \quad \overrightarrow{QR} = (6-5)\hat{i} + (5-3)\hat{j} = \hat{i} + 2\hat{j}$

2. Position vector of $P = 2\hat{i} + 2\hat{j}$

and position vector of $S = 9\hat{i} + 8\hat{j}$

$\therefore \quad \overrightarrow{PS} = (9-2)\hat{i} + (8-2)\hat{j} = 7\hat{i} + 6\hat{j}$

Now, $\quad |\overrightarrow{PS}|^2 = (7)^2 + (6)^2 = 49 + 36 = 85$

$\Rightarrow \quad |\overrightarrow{PS}| = \sqrt{85}$ units

3. $\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP} = (6\hat{i} + 5\hat{j}) - (2\hat{i} + 2\hat{j})$

$= (4\hat{i} + 3\hat{j})$

$\therefore$ Unit vector of $\overrightarrow{PR}$ or $\hat{PR} = \dfrac{\overrightarrow{PR}}{|\overrightarrow{PR}|}$

$$= \frac{4\hat{i} + 3\hat{j}}{|4\hat{i} + 3\hat{j}|} = \frac{4\hat{i} + 3\hat{j}}{\sqrt{16+9}}$$

$$= \frac{4\hat{i} + 3\hat{j}}{\sqrt{25}} = \frac{1}{5}(4\hat{i} + 3\hat{j})$$

Or

$\overrightarrow{QS} = \overrightarrow{OS} - \overrightarrow{OQ} = (9\hat{i} + 8\hat{j}) - (5\hat{i} + 3\hat{j})$

$= 4\hat{i} + 5\hat{j}$

and $\quad \overrightarrow{PR} = 4\hat{i} + 3\hat{j}$

$$\therefore \quad \overrightarrow{PR} \times \overrightarrow{QS} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 0 \\ 4 & 5 & 0 \end{vmatrix}$$

$$= (0-0)\hat{i} - (0-0)\hat{j} + (20-12)\hat{k} = 8\hat{k}$$

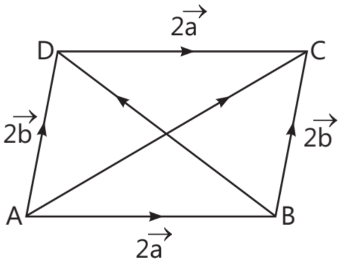
Case Study 8

If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the third side of the triangle taken in opposite order and this is known as triangle law of vector addition.

Based on the above information, solve the following questions:

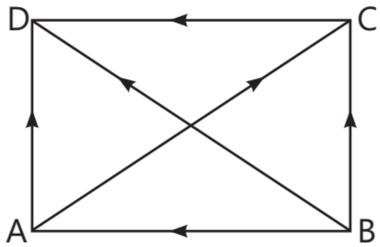
Q 1. If *ABCD* is a parallelogram and *AC* and *BD* are its diagonals, then find the value of $\overrightarrow{AC} + \overrightarrow{BD}$.

Q 2. If *ABCD* is a parallelogram, where $\overrightarrow{AB} = 2\vec{a}$ and $\overrightarrow{BC} = 2\vec{b}$, then find the value of $\overrightarrow{AC} - \overrightarrow{BD}$.

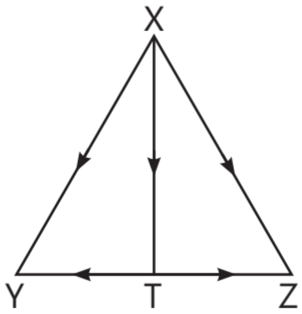


Or

If *ABCD* is a quadrilateral, whose diagonals are $\overrightarrow{AC}$ and $\overrightarrow{BD}$, then find the value of $\overrightarrow{BA} + \overrightarrow{CD}$.

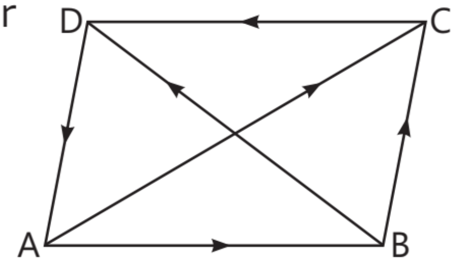


Q 3. If T is the mid-point of side YZ of ΔXYZ, then find the value of $\overrightarrow{XY} + \overrightarrow{XZ}$.



Solutions

1. From triangle law of vector addition,



$\overrightarrow{AC} + \overrightarrow{BD}$

$= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{BC} + \overrightarrow{CD}$

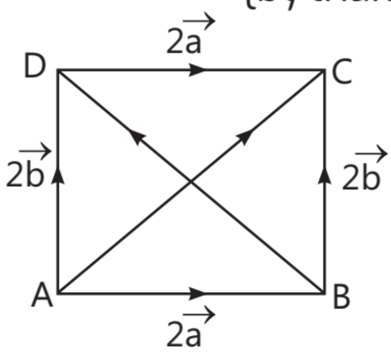
$= \overrightarrow{AB} + 2\,\overrightarrow{BC} + \overrightarrow{CD}$

$= \overrightarrow{AB} + 2\,\overrightarrow{BC} - \overrightarrow{AB}$

$= 2\,\overrightarrow{BC}$ $\qquad [\because \overrightarrow{AB} = -\overrightarrow{CD}]$

2. In ΔABC, $\overrightarrow{AC} = 2\vec{a} + 2\vec{b}$ ...(1)

[by triangle law of addition]



and in ΔABD, $2\vec{b} = 2\vec{a} + \overrightarrow{BD}$ ...(2)

[by triangle law of addition]

Adding eqs. (1) and (2), we have

$$\overrightarrow{AC} + 2\vec{b} = 4\vec{a} + \overrightarrow{BD} + 2\vec{b}$$

$$\Rightarrow \qquad \overrightarrow{AC} - \overrightarrow{BD} = 4\vec{a}$$

Or

In ΔABC, $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BC}$ [by triangle law] ...(1)

In ΔBCD, $\overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{BD}$ [by triangle law] ...(2)

From eqs. (1) and (2), $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BD} - \overrightarrow{CD}$

$$\Rightarrow \qquad \overrightarrow{BA} + \overrightarrow{CD} = \overrightarrow{BD} - \overrightarrow{AC} = \overrightarrow{BD} + \overrightarrow{CA}$$

3. Since, T is the mid-point of YZ.

So, $\overrightarrow{YT} = \overrightarrow{TZ}$

Now, $\overrightarrow{XY} + \overrightarrow{XZ} = (\overrightarrow{XT} + \overrightarrow{TY}) + (\overrightarrow{XT} + \overrightarrow{TZ})$

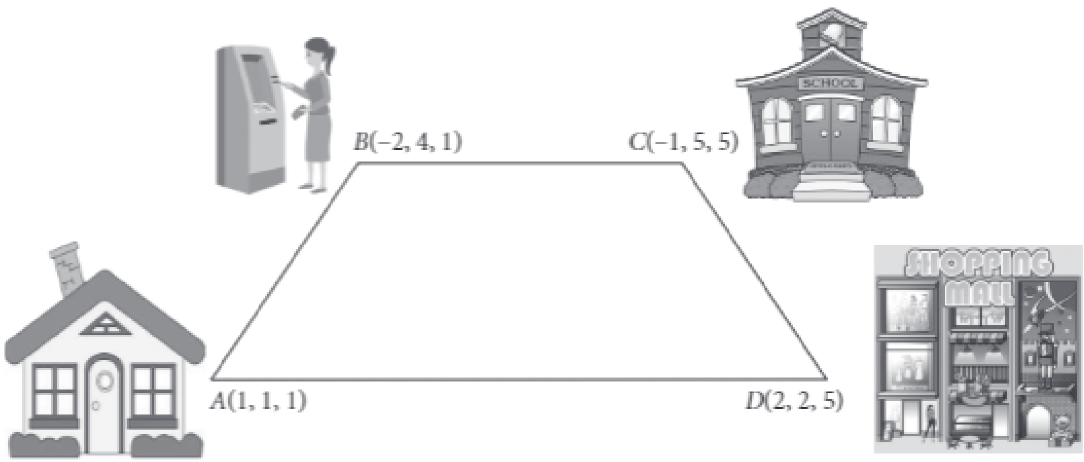
[by triangle law]

$$= 2\overrightarrow{XT} + \overrightarrow{TY} + \overrightarrow{TZ} = 2\overrightarrow{XT} \quad [\because \overrightarrow{TY} = -\overrightarrow{YT}]$$

Solutions for Questions 9 to 18 are Given Below

Case Study 9

Ritika starts walking from his house to shopping mall. Instead of going to the mall directly, she first goes to a ATM, from there to her daughter's school and then reaches the mall. In the diagram, *A*, *B*, *C* and *D* represent the coordinates of House, ATM, School and Mall respectively.



Based on the above information, answer the following questions.

(i) Distance between House (*A*) and ATM (*B*) is

(a) 3 units (b) $3\sqrt{2}$ units (c) $\sqrt{2}$ units (d) $4\sqrt{2}$ units

(ii) Distance between ATM (*B*) and School (*C*) is

(a) $\sqrt{2}$ units (b) $2\sqrt{2}$ units (c) $3\sqrt{2}$ units (d) $4\sqrt{2}$ units

(iii) Distance between School (*C*) and Shopping mall (*D*) is

(a) $3\sqrt{2}$ units (b) $5\sqrt{2}$ units (c) $7\sqrt{2}$ units (d) $10\sqrt{2}$ units

(iv) What is the total distance travelled by Ritika?

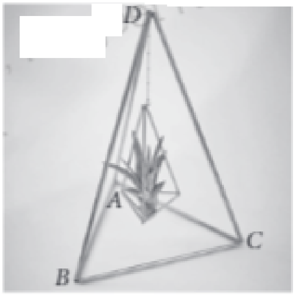
(a) $4\sqrt{2}$ units (b) $6\sqrt{2}$ units (c) $8\sqrt{2}$ units (d) $9\sqrt{2}$ units

(v) What is the extra distance travelled by Ritika in reaching the shopping mall?

(a) $3\sqrt{2}$ units (b) $5\sqrt{2}$ units (c) $6\sqrt{2}$ units (d) $7\sqrt{2}$ units

Case Study 10

Ginni purchased an air plant holder which is in the shape of a tetrahedron.
Let A, B, C and D are the coordinates of the air plant holder where $A \equiv (1, 1, 1)$, $B \equiv (2, 1, 3)$, $C \equiv (3, 2, 2)$ and $D \equiv (3, 3, 4)$.



Based on the above information, answer the following questions.

(i) Find the position vector of $\overrightarrow{AB}$.

(a) $-\hat{i} - 2\hat{k}$ (b) $2\hat{i} + \hat{k}$ (c) $\hat{i} + 2\hat{k}$ (d) $-2\hat{i} - \hat{k}$

(ii) Find the position vector of $\overrightarrow{AC}$.

(a) $2\hat{i} - \hat{j} - \hat{k}$ (b) $2\hat{i} + \hat{j} + \hat{k}$ (c) $-2\hat{i} - \hat{j} + \hat{k}$ (d) $\hat{i} + 2\hat{j} + \hat{k}$

(iii) Find the position vector of $\overrightarrow{AD}$.

(a) $2\hat{i} - 2\hat{j} - 3\hat{k}$ (b) $\hat{i} + \hat{j} - 3\hat{k}$ (c) $3\hat{i} + 2\hat{j} + 2\hat{k}$ (d) $2\hat{i} + 2\hat{j} + 3\hat{k}$

(iv) Area of $\Delta ABC =$

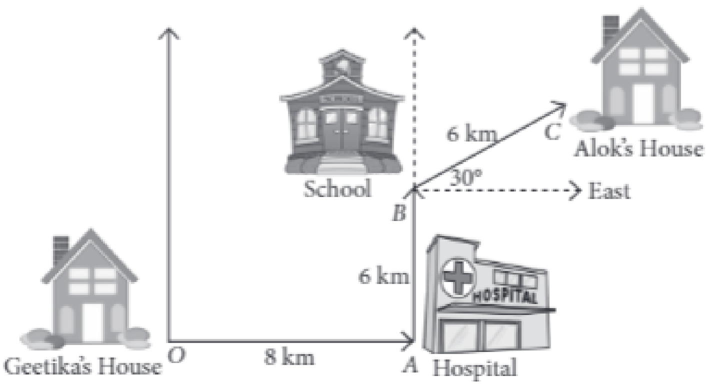
(a) $\frac{\sqrt{11}}{2}$ sq. units (b) $\frac{\sqrt{14}}{2}$ sq. units (c) $\frac{\sqrt{13}}{2}$ (d) $\frac{\sqrt{17}}{2}$ sq. units

(v) Find the unit vector along $\overrightarrow{AD}$.

(a) $\frac{1}{\sqrt{17}}(2\hat{i} + 2\hat{j} + 3\hat{k})$ (b) $\frac{1}{\sqrt{17}}(3\hat{i} + 3\hat{j} + 2\hat{k})$ (c) $\frac{1}{\sqrt{11}}(2\hat{i} + 2\hat{j} + 3\hat{k})$ (d) $(2\hat{i} + 2\hat{j} + 3\hat{k})$

Case Study 11

Geetika's house is situated at Shalimar Bagh at point O, for going to Alok's house she first travels 8 km by bus in the East. Here at point A, a hospital is situated. From Hospital, Geetika takes an auto and goes 6 km in the North, here at point B school is situated. From school, she travels by bus to reach Alok's house which is at 30° East, 6 km from point B.



Based on the above information, answer the following questions.

(i) What is the vector distance between Geetika's house and school?

(a) $8\hat{i} - 6\hat{j}$ (b) $8\hat{i} + 6\hat{j}$ (c) $8\hat{i}$ (d) $6\hat{j}$

(ii) How much distance Geetika travels to reach school?

(a) 14 km (b) 15 km (c) 16 km (d) 17 km

(iii) What is the vector distance from school to Alok's house?

(a) $\sqrt{3}\,\hat{i} + \hat{j}$ (b) $3\sqrt{3}\,\hat{i} + 3\hat{j}$ (c) $6\hat{i}$ (d) $6\hat{j}$

(iv) What is the vector distance from Geetika's house to Alok's house?

(a) $(8 + 3\sqrt{3})\hat{i} + 9\hat{j}$ (b) $4\hat{i} + 6\hat{j}$ (c) $15\hat{i}$ (d) $16\hat{j}$

(v) What is the total distance travelled by Geetika from her house to Alok's house?

(a) 19 km (b) 20 km (c) 21 km (d) 22 km

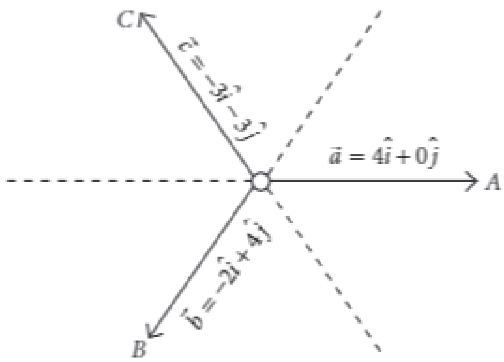
Case Study 12

Teams *A*, *B*, *C* went for playing a tug of war game. Teams *A*, *B*, *C* have attached a rope to a metal ring and is trying to pull the ring into their own area (team areas shown below).

Team *A* pulls with force $F_1 = 4\hat{i} + 0\hat{j}$ KN

Team $B \rightarrow F_2 = -2\hat{i} + 4\hat{j}$ KN

Team $C \rightarrow F_3 = -3\hat{i} - 3\hat{j}$ KN



Based on the above information, answer the following questions.

(i) Which team will win the game?

(a) Team *B* (b) Team *A* (c) Team *C* (d) No one

(ii) What is the magnitude of the teams combined force?

(a) 7 KN (b) 1.4 KN (c) 1.5 KN (d) 2 KN

(iii) In what direction is the ring getting pulled?

(a) 2.0 radian (b) 2.5 radian (c) 2.4 radian (d) 3 radian

(iv) What is the magnitude of the force of Team *B*?

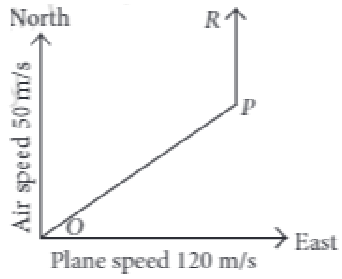
(a) $2\sqrt{5}$ KN (b) 6 KN (c) 2 KN (d) $\sqrt{6}$ KN

(v) How many KN force is applied by Team *A*?

(a) 5 KN (b) 4 KN (c) 2 KN (d) 16 KN

Case Study 13

A plane started from airport situated at O with a velocity of 120 m/s towards east. Air is blowing at a velocity of 50 m/s towards the north as shown in the figure.
The plane travelled 1 hr in OP direction with the resultant velocity. From P to R the plane travelled 1 hr keeping velocity of 120 m/s and finally landed at R.



Based on the above information, answer the following questions.

(i) What is the resultant velocity from O to P?
(a) 100 m/s (b) 130 m/s (c) 126 m/s (d) 180 m/s

(ii) What is the direction of travel of plane from O to P with East?
(a) $\tan^{-1}\left(\frac{5}{12}\right)$ (b) $\tan^{-1}\left(\frac{12}{3}\right)$ (c) 50 (d) 80

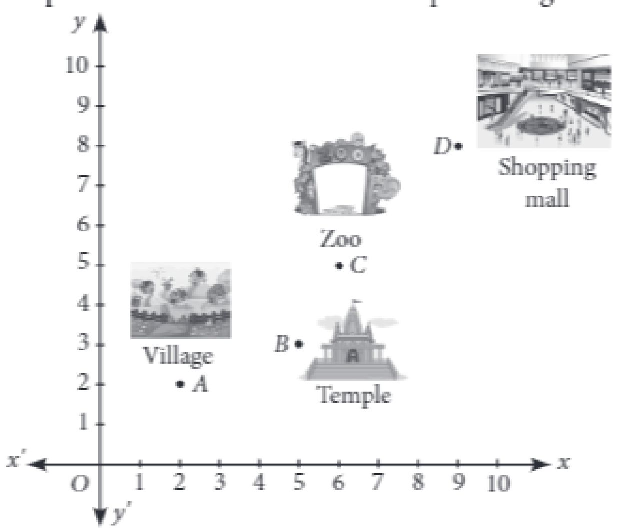
(iii) What is the displacement from O to P?
(a) 600 km (b) 468 km (c) 532 km (d) 500 km

(iv) What is the resultant velocity from P to R?
(a) 120 m/s (b) 70 m/s (c) 170 m/s (d) 200 m/s

(v) What is the displacement from P to R?
(a) 450 km (b) 532 km (c) 610 km (d) 612 km

Case Study 14

Ishaan left from his village on weekend. First, he travelled up to temple. After this, he left for the zoo. After this he left for shopping in a mall. The positions of Ishaan at different places is given in the following graph.



Based on the above information, answer the following questions.

(i) Position vector of B is

(a) $3\hat{i}+5\hat{j}$ (b) $5\hat{i}+3\hat{j}$ (c) $-5\hat{i}-3\hat{j}$ (d) $-5\hat{i}+3\hat{j}$

(ii) Position vector of D is

(a) $5\hat{i}+3\hat{j}$ (b) $3\hat{i}+5\hat{j}$ (c) $8\hat{i}+9\hat{j}$ (d) $9\hat{i}+8\hat{j}$

(iii) Find the vector $\overrightarrow{BC}$ in terms of $\hat{i}, \hat{j}$.

(a) $\hat{i}-2\hat{j}$ (b) $\hat{i}+2\hat{j}$ (c) $2\hat{i}+\hat{j}$ (d) $2\hat{i}-\hat{j}$

(iv) Length of vector $\overrightarrow{AD}$ is

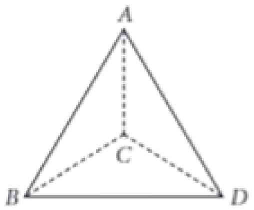
(a) $\sqrt{67}$ units (b) $\sqrt{85}$ units (c) 90 units (d) 100 units

(v) If $\vec{M}=4\hat{j}+3\hat{k}$, then its unit vector is

(a) $\frac{4}{5}\hat{j}+\frac{3}{5}\hat{k}$ (b) $\frac{4}{5}\hat{j}-\frac{3}{5}\hat{k}$ (c) $-\frac{4}{5}\hat{j}+\frac{3}{5}\hat{k}$ (d) $-\frac{4}{5}\hat{j}-\frac{3}{5}\hat{k}$

Case Study 15

A building is to be constructed in the form of a triangular pyramid, $ABCD$ as shown in the figure.



Let its angular points are $A(0, 1, 2)$, $B(3, 0, 1)$, $C(4, 3, 6)$ and $D(2, 3, 2)$ and G be the point of intersection of the medians of ΔBCD.

Based on the above information, answer the following questions.

(i) The coordinates of point G are

(a) (2, 3, 3) (b) (3, 3, 2) (c) (3, 2, 3) (d) (0, 2, 3)

(ii) The length of vector $\overrightarrow{AG}$ is

(a) $\sqrt{17}$ units (b) $\sqrt{11}$ units (c) $\sqrt{13}$ units (d) $\sqrt{19}$ units

(iii) Area of ΔABC (in sq. units) is

(a) $\sqrt{10}$ (b) $2\sqrt{10}$ (c) $3\sqrt{10}$ (d) $5\sqrt{10}$

(iv) The sum of lengths of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is

(a) 5 units (b) 9.32 units (c) 10 units (d) 11 units

(v) The length of the perpendicular from the vertex D on the opposite face is

(a) $\frac{6}{\sqrt{10}}$ units (b) $\frac{2}{\sqrt{10}}$ units (c) $\frac{3}{\sqrt{10}}$ units (d) $8\sqrt{10}$ units

Case Study 16

If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the third side of the triangle taken in opposite order and this is known as triangle law of vector addition.
Based on the above information, answer the following questions.

(i) If $\vec{p}, \vec{q}, \vec{r}$ are the vectors represented by the sides of a triangle taken in order, then $\vec{q} + \vec{r} =$

(a) $\vec{p}$ (b) $2\vec{p}$ (c) $-\vec{p}$ (d) None of these

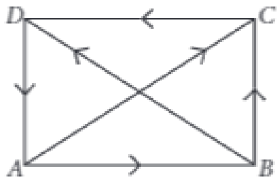
(ii) If $ABCD$ is a parallelogram and AC and BD are its diagonals, then $\overrightarrow{AC} + \overrightarrow{BD} =$

(a) $2\overrightarrow{DA}$ (b) $2\overrightarrow{AB}$ (c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{BD}$

(iii) If $ABCD$ is a parallelogram, where $\overrightarrow{AB} = 2\vec{a}$ and $\overrightarrow{BC} = 2\vec{b}$, then $\overrightarrow{AC} - \overrightarrow{BD} =$

(a) $3\vec{a}$ (b) $4\vec{a}$ (c) $2\vec{b}$ (d) $4\vec{b}$

(iv) If $ABCD$ is a quadrilateral whose diagonals are $\overrightarrow{AC}$ and $\overrightarrow{BD}$, then $\overrightarrow{BA} + \overrightarrow{CD} =$



(a) $\overrightarrow{AC} + \overrightarrow{DB}$ (b) $\overrightarrow{AC} + \overrightarrow{BD}$ (c) $\overrightarrow{BC} + \overrightarrow{AD}$ (d) $\overrightarrow{BD} + \overrightarrow{CA}$

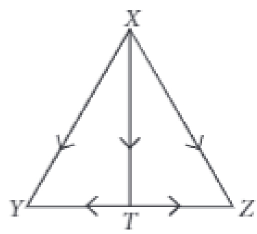
(v) If T is the mid point of side YZ of ΔXYZ, then $\overrightarrow{XY} + \overrightarrow{XZ} =$

(a) $2\overrightarrow{YT}$

(b) $2\overrightarrow{XT}$

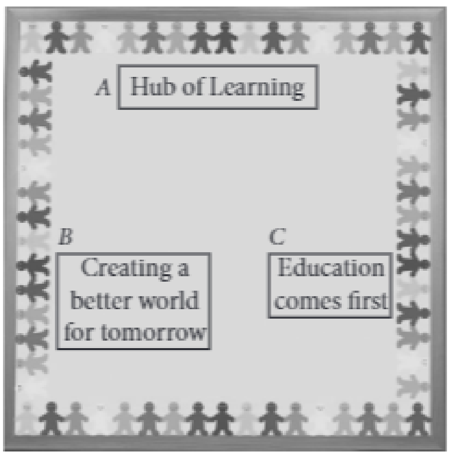
(c) $2\overrightarrow{TZ}$

(d) None of these



Case Study 17

Three slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (Hub of Learning), B (Creating a better world for tomorrow) and C (Education comes first). The coordinates of these points are (1, 4, 2), (3, –3, –2) and (–2, 2, 6) respectively.



Based on the above information, answer the following questions.

(i) Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the position vectors of points A, B and C respectively, then $\vec{a}+\vec{b}+\vec{c}$ is equal to

(a) $2\hat{i}+3\hat{j}+6\hat{k}$ (b) $2\hat{i}-3\hat{j}-6\hat{k}$ (c) $2\hat{i}+8\hat{j}+3\hat{k}$ (d) $2(7\hat{i}+8\hat{j}+3\hat{k})$

(ii) Which of the following is not true?

(a) $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$ (b) $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{AC}=\vec{0}$ (c) $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{CA}=\vec{0}$ (d) $\overrightarrow{AB}-\overrightarrow{CB}+\overrightarrow{CA}=\vec{0}$

(iii) Area of ΔABC is

(a) 19 sq. units (b) $\sqrt{1937}$ sq. units (c) $\frac{1}{2}\sqrt{1937}$ sq. units (d) $\sqrt{1837}$ sq. units

(iv) Suppose, if the given slogans are to be placed on a straight line, then the value of $|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|$ will be equal to

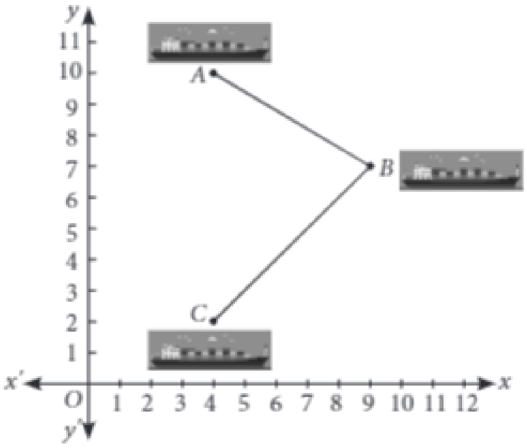
(a) –1 (b) –2 (c) 2 (d) 0

(v) If $\vec{a}=2\hat{i}+3\hat{j}+6\hat{k}$, then unit vector in the direction of vector $\vec{a}$ is

(a) $\frac{2}{7}\hat{i}-\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}$ (b) $\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{6}{7}\hat{k}$ (c) $\frac{3}{7}\hat{i}+\frac{2}{7}\hat{j}+\frac{6}{7}\hat{k}$ (d) None of these

Case Study 18

A barge is pulled into harbour by two tug boats as shown in the figure.



Based on the above information, answer the following questions.

(i) Position vector of A is

(a) $4\hat{i}+2\hat{j}$ (b) $4\hat{i}+10\hat{j}$ (c) $4\hat{i}-10\hat{j}$ (d) $4\hat{i}-2\hat{j}$

(ii) Position vector of B is

(a) $4\hat{i}+4\hat{j}$ (b) $6\hat{i}+6\hat{j}$ (c) $9\hat{i}+7\hat{j}$ (d) $3\hat{i}+3\hat{j}$

(iii) Find the vector $\overrightarrow{AC}$ in terms of $\hat{i}, \hat{j}$.

(a) $8\hat{j}$ (b) $-8\hat{j}$ (c) $8\hat{i}$ (d) None of these

(iv) If $\vec{A}=\hat{i}+2\hat{j}+3\hat{k}$, then its unit vector is

(a) $\frac{\hat{i}}{\sqrt{14}}+\frac{2\hat{j}}{\sqrt{14}}+\frac{3\hat{k}}{\sqrt{14}}$ (b) $\frac{3\hat{i}}{\sqrt{14}}+\frac{2\hat{j}}{\sqrt{14}}+\frac{\hat{k}}{\sqrt{14}}$ (c) $\frac{2\hat{i}}{\sqrt{14}}+\frac{3\hat{j}}{\sqrt{14}}+\frac{\hat{k}}{\sqrt{14}}$ (d) None of these

(v) If $\vec{A}=4\hat{i}+3\hat{j}$ and $\vec{B}=3\hat{i}+4\hat{j}$, then $|\vec{A}|+|\vec{B}|=$ ______.

(a) 12 (b) 13 (c) 14 (d) 10

9. (i) (b): $\overrightarrow{AB} = (-2\hat{i} + 4\hat{j} + \hat{k}) - (\hat{i} + \hat{j} + \hat{k}) = -3\hat{i} + 3\hat{j}$

$\therefore \quad \overrightarrow{AB} = \sqrt{(-3)^2 + 3^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

Distance between House (A) and ATM (B) is $3\sqrt{2}$ units.

(ii) (c): $\overrightarrow{BC} = (-\hat{i} + 5\hat{j} + 5\hat{k}) - (-2\hat{i} + 4\hat{j} + \hat{k}) = \hat{i} + \hat{j} + 4\hat{k}$

$\therefore \quad |\overrightarrow{BC}| = \sqrt{1^2 + 1^2 + 4^2} = \sqrt{1+1+16}$

$= \sqrt{18} = 3\sqrt{2}$

Distance between ATM (B) and School (C) is $3\sqrt{2}$ units.

(iii) (a): $\overrightarrow{CD} = (2\hat{i} + 2\hat{j} + 5\hat{k}) - (-\hat{i} + 5\hat{j} + 5\hat{k}) = 3\hat{i} - 3\hat{j}$

$\therefore \quad |\overrightarrow{CD}| = \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = 3\sqrt{2}$

Distance between School (C) and Shopping mall (D) is $3\sqrt{2}$ units.

(iv) (d): Total distance travelled by Ritika

$= |\overrightarrow{AB}| + |\overrightarrow{BC}| + |\overrightarrow{CD}| = (3\sqrt{2} + 3\sqrt{2} + 3\sqrt{2})$ units

$= 9\sqrt{2}$ units

(v) (c): Distance between house and shopping mall is $|\overrightarrow{AD}|$.

Now, $\overrightarrow{AD} = \hat{i} + \hat{j} + 4\hat{k}$

$\therefore \quad |\overrightarrow{AD}| = \sqrt{1^2 + 1^2 + 4^2} = \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$

Thus, extra distance travelled by Ritika in reaching shopping mall $= (9\sqrt{2} - 3\sqrt{2})$ units $= 6\sqrt{2}$ units

10. (i) (c): Position vector of $\overrightarrow{AB}$

$= (2-1)\hat{i} + (1-1)\hat{j} + (3-1)\hat{k} = \hat{i} + 2\hat{k}$

(ii) (b): Position vector of $\overrightarrow{AC}$

$= (3-1)\hat{i} + (2-1)\hat{j} + (2-1)\hat{k} = 2\hat{i} + \hat{j} + \hat{k}$

(iii) (d): Position vector of $\overrightarrow{AD}$

$= (3-1)\hat{i} + (3-1)\hat{j} + (4-1)\hat{k} = 2\hat{i} + 2\hat{j} + 3\hat{k}$

(iv) (b): Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ 2 & 1 & 1 \end{vmatrix} = \hat{i}(0-2) - \hat{j}(1-4) + \hat{k}(1-0)$$

$= -2\hat{i} + 3\hat{j} + \hat{k}$

$\Rightarrow \quad |\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-2)^2 + 3^2 + 1^2}$

$= \sqrt{4+9+1} = \sqrt{14}$

$\therefore \quad$ Area of $\Delta ABC = \frac{1}{2}\sqrt{14}$ sq. units

(v) (a): Unit vector along $\overrightarrow{AD} = \frac{\overrightarrow{AD}}{|\overrightarrow{AD}|}$

$= \frac{2\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{2^2 + 2^2 + 3^2}} = \frac{2\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{4+4+9}} = \frac{1}{\sqrt{17}}(2\hat{i} + 2\hat{j} + 3\hat{k})$

11. (i) (b): We have, $\overrightarrow{OA} = 8\hat{i}$ and $\overrightarrow{AB} = 6\hat{j}$

$\therefore \quad \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = 8\hat{i} + 6\hat{j}$

(ii) (a): To reach school Geetika travels

$= (8 + 6)$ km $= 14$ km

(iii) (b): Vector distance from school to Alok's house

$= 6\cos 30^\circ \hat{i} + 6\sin 30^\circ \hat{j}$

$= 6 \times \frac{\sqrt{3}}{2}\hat{i} + 6 \times \frac{1}{2}\hat{j} = 3\sqrt{3}\hat{i} + 3\hat{j}$

(iv) (a): Vector distance from Geetika's house to Alok's house $= 8\hat{i} + 6\hat{j} + 3\sqrt{3}\hat{i} + 3\hat{j} = (8 + 3\sqrt{3})\hat{i} + 9\hat{j}$

(v) (b): Total distance travelled by Geetika from her house to Alok's house $= (8 + 6 + 6)$ km $= 20$ km

12. Here, $|\vec{F_1}| = \sqrt{(4)^2 + 0^2} = 4$ KN

$|\vec{F_2}| = \sqrt{(-2)^2 + 4^2} = \sqrt{20}$ KN

$|\vec{F_3}| = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18}$ KN

(i) (a): Since, $\sqrt{20}$ is larger. So, team B will win the game.

(ii) (b): Let $\vec{F}$ be the combined force.

$\therefore \quad \vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} = 4\hat{i} + 0\hat{j} - 3\hat{i} - 3\hat{j} - 2\hat{i} + 4\hat{j}$

$= -\hat{i} + \hat{j}$

$\therefore \quad |\vec{F}| = \sqrt{(-1)^2 + 1^2} = \sqrt{2} = 1.4$ KN

(iii) (c): We have, $\vec{F} = -\hat{i} + \hat{j}$

$\therefore \quad \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{1}{-1}\right) = \frac{3\pi}{4}$ radian

$= 0.75 \times 3.14$ radian $= 2.3555$ radian ≈ 2.4 radian

(iv) (a): Magnitude of force of Team $B = \sqrt{20}$ KN

$= 2\sqrt{5}$ KN

(v) (b): 4 KN force is applied by team A.

13. (i) (b): Resultant velocity from to O to P

$= \sqrt{120^2 + 50^2}$

$= \sqrt{14400 + 2500} = \sqrt{16900} = 130$ m/s

(ii) (a): Direction of travel of plane from O to P with east is $\tan^{-1}\left(\frac{5}{12}\right)$.

(iii) (b): Resultant velocity from O to A = 130 m/s

$= \left(\frac{130 \times 3600}{1000}\right)$ km/h

Time = 1 hr

$\therefore$ Displacement $= \frac{130 \times 3600}{1000} = 468$ km

(iv) (c): Resultant velocity from P to R

$= (120 + 50) = 170$ m/s

(v) (d): Displacement from P to $R = \left(\frac{170 \times 3600}{1000}\right)$

$= 612$ km

14. (i) (b): Here (5, 3) are the coordinates of B.

$\therefore$ P.V. of $B = 5\hat{i} + 3\hat{j}$

(ii) (d) : Here (9, 8) are the coordinates of D.

$\therefore$ P.V. of $D = 9\hat{i} + 8\hat{j}$

(iii) (b): P.V. of $B = 5\hat{i} + 3\hat{j}$ and P.V. of $C = 6\hat{i} + 5\hat{j}$

$\therefore \overrightarrow{BC} = (6-5)\hat{i} + (5-3)\hat{j} = \hat{i} + 2\hat{j}$

(iv) (b): Since P.V. of $A = 2\hat{i} + 2\hat{j}$, P.V. of $D = 9\hat{i} + 8\hat{j}$

$\therefore \overrightarrow{AD} = (9-2)\hat{i} + (8-2)\hat{j} = 7\hat{i} + 6\hat{j}$

$|\overrightarrow{AD}|^2 = 7^2 + 6^2 = 49 + 36 = 85$

$\Rightarrow |\overrightarrow{AD}| = \sqrt{85}$ units

(v) (a): We have, $\vec{M} = 4\hat{j} + 3\hat{k}$,

$\therefore |\vec{M}| = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$

$\therefore \hat{M} = \frac{\vec{M}}{|\vec{M}|} = \frac{4\hat{j} + 3\hat{k}}{5} = \frac{4}{5}\hat{j} + \frac{3}{5}\hat{k}$

15. (i) (c): Clearly, G be the centroid of ΔBCD, therefore coordinates of G are

$\left(\frac{3+4+2}{3}, \frac{0+3+3}{3}, \frac{1+6+2}{3}\right) = (3, 2, 3)$

(ii) (b): Since, $A \equiv (0, 1, 2)$ and $G = (3, 2, 3)$

$\therefore \overrightarrow{AG} = (3-0)\hat{i} + (2-1)\hat{j} + (3-2)\hat{k} = 3\hat{i} + \hat{j} + \hat{k}$

$\Rightarrow |\overrightarrow{AG}|^2 = 3^2 + 1^2 + 1^2 = 9 + 1 + 1 = 11$

$\Rightarrow |\overrightarrow{AG}| = \sqrt{11}$

(iii) (c): Clearly, area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

Here, $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3-0 & 0-1 & 1-2 \\ 4-0 & 3-1 & 6-2 \end{vmatrix}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -1 \\ 4 & 2 & 4 \end{vmatrix}$

$= \hat{i}(-4+2) - \hat{j}(12+4) + \hat{k}(6+4) = -2\hat{i} - 16\hat{j} + 10\hat{k}$

$\therefore |\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-2)^2 + (-16)^2 + 10^2}$

$= \sqrt{4 + 256 + 100} = \sqrt{360} = 6\sqrt{10}$

Hence, area of $\Delta ABC = \frac{1}{2} \times 6\sqrt{10} = 3\sqrt{10}$ sq. units

(iv) (b): Here, $\overrightarrow{AB} = 3\hat{i} - \hat{j} - \hat{k}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{9+1+1} = \sqrt{11}$

Also, $\overrightarrow{AC} = 4\hat{i} + 2\hat{j} + 4\hat{k}$

$\Rightarrow |\overrightarrow{AC}| = \sqrt{16+4+16} = \sqrt{36} = 6$

Now, $|\overrightarrow{AB}| + |\overrightarrow{AC}| = \sqrt{11} + 6 = 3.32 + 6 = 9.32$ units

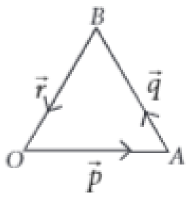
(v) (a): The length of the perpendicular from the vertex D on the opposite face

$= |$Projection of $\overrightarrow{AD}$ on $\overrightarrow{AB} \times \overrightarrow{AC}|$

$= \left|\frac{(2\hat{i} + 2\hat{j}) \cdot (-2\hat{i} - 16\hat{j} + 10\hat{k})}{\sqrt{(-2)^2 + (-16)^2 + 10^2}}\right|$

$= \left|\frac{-4-32}{\sqrt{360}}\right| = \frac{36}{6\sqrt{10}} = \frac{6}{\sqrt{10}}$ units

16. (i) (c): Let OAB be a triangle such that



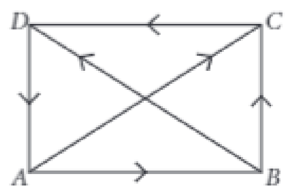
$\overrightarrow{AO} = -\vec{p}, \overrightarrow{AB} = \vec{q}, \overrightarrow{BO} = \vec{r}$

Now, $\vec{q} + \vec{r} = \overrightarrow{AB} + \overrightarrow{BO}$

$= \overrightarrow{AO} = -\vec{p}$

(ii) (c) : From triangle law of vector addition,

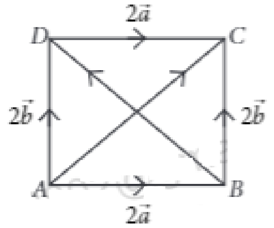
$\overrightarrow{AC} + \overrightarrow{BD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{BC} + \overrightarrow{CD}$



$= \overrightarrow{AB} + 2\overrightarrow{BC} + \overrightarrow{CD}$

$= \overrightarrow{AB} + 2\overrightarrow{BC} - \overrightarrow{AB} = 2\overrightarrow{BC}$ $\quad [\because \overrightarrow{AB} = -\overrightarrow{CD}]$

(iii) (b): In ΔABC, $\overrightarrow{AC} = 2\vec{a} + 2\vec{b}$ ...(i)



and in ΔABD, $2\vec{b} = 2\vec{a} + \overrightarrow{BD}$ ...(ii)

[By triangle law of addition]

Adding (i) and (ii), we have

$\overrightarrow{AC} + 2\vec{b} = 4\vec{a} + \overrightarrow{BD} + 2\vec{b}$

$\Rightarrow \overrightarrow{AC} - \overrightarrow{BD} = 4\vec{a}$

(iv) (d): In ΔABC, $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BC}$ ...(i)

[By triangle law]

In ΔBCD, $\overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{BD}$ ...(ii)

From (i) and (ii), $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BD} - \overrightarrow{CD}$

$\Rightarrow \overrightarrow{BA} + \overrightarrow{CD} = \overrightarrow{BD} - \overrightarrow{AC} = \overrightarrow{BD} + \overrightarrow{CA}$

(v) (b): Since T is the mid point of YZ.

So, $\overrightarrow{YT} = \overrightarrow{TZ}$

Now, $\overrightarrow{XY} + \overrightarrow{XZ} = (\overrightarrow{XT} + \overrightarrow{TY}) + (\overrightarrow{XT} + \overrightarrow{TZ})$

[By triangle law]

Now, $\overrightarrow{XY} + \overrightarrow{XZ} = (\overrightarrow{XT} + \overrightarrow{TY}) + (\overrightarrow{XT} + \overrightarrow{TZ})$

[By triangle law]

$= 2\overrightarrow{XT} + \overrightarrow{TY} + \overrightarrow{TZ} = 2\overrightarrow{XT}$ $\quad [\because \overrightarrow{TY} = -\overrightarrow{YT}]$

17. **(i) (a):** $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 3\hat{j} - 2\hat{k}$

and $\vec{c} = -2\hat{i} + 2\hat{j} + 6\hat{k}$

$\therefore \quad \vec{a} + \vec{b} + \vec{c} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

(ii) (c) : Using triangle law of addition in ΔABC, we get

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$, which can be rewritten as

$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$ or $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

(iii) (c) : We have, $A(1, 4, 2)$, $B(3, -3, -2)$ and $C(-2, 2, 6)$

Now, $\overrightarrow{AB} = \vec{b} - \vec{a} = 2\hat{i} - 7\hat{j} - 4\hat{k}$

and $\overrightarrow{AC} = \vec{c} - \vec{a} = -3\hat{i} - 2\hat{j} + 4\hat{k}$

$$\therefore \quad \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & -4 \\ -3 & -2 & 4 \end{vmatrix}$$

$= \hat{i}(-28 - 8) - \hat{j}(8 - 12) + \hat{k}(-4 - 21) = -36\hat{i} + 4\hat{j} - 25\hat{k}$

Now, $|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-36)^2 + 4^2 + (-25)^2}$

$= \sqrt{1296 + 16 + 625} = \sqrt{1937}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{1937}$ sq. units

(iv) (d): If the given points lie on the straight line, then the points will be collinear and so area of $\Delta ABC = 0$.

$\Rightarrow |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}| = 0$

[$\because$ If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the three vertices A, B and C of ΔABC, then area of triangle $= \frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$]

(v) (b): Here, $|\vec{a}| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36}$

$= \sqrt{49} = 7$

$\therefore$ Unit vector in the direction of vector $\vec{a}$ is

$$\hat{a} = \frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{7} = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$

18. **(i) (b):** Here, (4, 10) are the coordinates of A.

$\therefore$ P.V. of $A = 4\hat{i} + 10\hat{j}$

(ii) (c) : Here, (9, 7) are the coordinates of B.

$\therefore$ P.V. of $B = 9\hat{i} + 7\hat{j}$

(ii) (c) : Here, (9, 7) are the coordinates of B.

$\therefore$ P.V. of $B = 9\hat{i} + 7\hat{j}$

(iii) (b): Here, P.V. of $A = 4\hat{i} + 10\hat{j}$ and P.V. of $C = 4\hat{i} + 2\hat{j}$

$\therefore \quad \overrightarrow{AC} = (4 - 4)\hat{i} + (2 - 10)\hat{j} = -8\hat{j}$

(iv) (a): Here $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore \quad |\vec{A}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$

$$\therefore \quad \hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}} = \frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$$

(v) (d): We have, $\vec{A} = 4\hat{i} + 3\hat{j}$ and $\vec{B} = 3\hat{i} + 4\hat{j}$

$\therefore \quad |\vec{A}| = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

and $|\vec{B}| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

Thus, $|\vec{A}| + |\vec{B}| = 5 + 5 = 10$